Q: A 12.0-cm-diameter solenoid is wound with 1200 turns per meter. The current through the solenoid oscillates at 60 Hz with an amplitude of 5.0 A. What is the maximum strength of the induced electric field inside the solenoid?

A: Let:

$r = 6 \times 10^{-2}$ m
$n = 1200$ m^{-1}
$\omega = 2\pi f = 120\pi$ s^{-1}
$I_0 = 5.0$ A

We begin by noting that the area of the solenoid is

$$A = \pi r^2$$

and that the magnetic field in the center of the solenoid due to the current (as derived from Ampere's law) is given by

$$B = \mu n I(t) \text{ where } I(t) = I_0 \sin(\omega t).$$

(Here we assume that solenoid is long and filled with air so that $\mu = \mu_0 = 4\pi \times 10^{-7}$ H/m.)

Using Faraday's law of induction and the fact that $\Phi_B = AB$, we have

$$\mathcal{E} = \int \vec{E} \cdot \mathrm{d}\vec{\ell} = 2\pi r E = \frac{\partial \Phi_B}{\partial t}$$
$$2\pi r E = \pi \mu_0 n r^2 I_0 \frac{\partial}{\partial t} \sin(\omega t).$$

$$E = \frac{1}{2} \mu_0 n r I_0 \omega \cos(\omega t)$$

This is maximized when $\cos(\omega t) = 1$, so that the maximum field is

$$E_{\max} = \frac{1}{2} \mu_0 n r I_0 \omega = (0.5)(4\pi \times 10^{-7} \text{ H/m})(1200 \text{ m}^{-1})(6 \times 10^{-2} \text{ m})(5.0 \text{ A})(120\pi \text{ s}^{-1})$$

$$\boxed{E_{\max} \approx 8.5 \times 10^{-2} \text{ N/C}}$$